1

(1) $f(x-3) = (x-3)^3 + 3(x-3) - 2$

$= x^3 - 9x^2 + 27x - 27 + 3x - 9 - 2$

$= x^3 - 9x^2 + 30x - 38$

$(x-3)(x-3)(x-3)$

$(x^2 - 6x + 9)(x-3)$

$x^3 - 6x^2 + 9x - 3x^2 + 18x - 27$

$x^3 - 9x^2 + 27x - 27$

(2) $(f \circ h \circ g)(x)$

$g(x) = 2x^2$

$h(2x^2) = \sqrt{2x^2 - 9}$

$f(\sqrt{2x^2-9}) = \sqrt{2x^2-9} + 3$

(3) $\dfrac{(x+h)^2 - 2(x+h) + 5 - (x^2 - 2x + 5)}{h}$ $\quad h \neq 0$

$\dfrac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h \cancel{+5} \cancel{-x^2} \cancel{+2x} \cancel{-5}}{h} = \dfrac{2xh + h^2 - 2h}{h} = 2x + h - 2, \ h \neq 0$

(4) $f(x) = \sqrt{2x+3}$

$y = \sqrt{2x+3}$

$(x)^2 = (\sqrt{2y+3})^2$

$x^2 = 2y + 3$

$\dfrac{x^2 - 3}{2} = \dfrac{2y}{2}$

$\dfrac{x^2-3}{2} = y$

(5) $m = \dfrac{4-2}{4-(-7)} = \dfrac{2}{11}$

point slope $\quad y - 4 = \frac{2}{11}(x-4)$ or $y - 2 = \frac{2}{11}(x+7)$

$y - 4 = \frac{2}{11}x - \frac{8}{11}$

$y = \frac{2}{11}x - \frac{8}{11} + 4$

slope intercept $\quad y = \frac{2}{11}x + \frac{36}{11}$

$11\left(\frac{2}{11}x - y + \frac{36}{11} = 0\right)$

standard form $\quad 2x - 11y + 36 = 0$

(6) $f(x) = x + 12$

$g(x) = \sqrt{x}$

$h(x) = \dfrac{12}{x}$ as $h(g(f(x)))$

(7) $f(g(x)) = g(f(x)) = x$

$f(x^2-3)$ $\quad\quad$ $g(\sqrt{x+3})$

$\sqrt{x^2 \cancel{-3} \cancel{+3}}$ $\quad\quad$ $(\sqrt{x+3})^2 - 3$

$\sqrt{x^2}$ $\quad\quad$ $x \cancel{+3} \cancel{-3}$ $\quad\quad$ $\therefore$ f and g are inverses

$x \quad = \quad x$

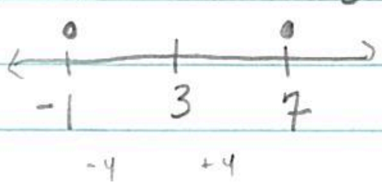
(8) $\dfrac{2x^4}{x^3-x^2} = \dfrac{2\cancel{x^4}^{\,2}}{\cancel{x^2}(x-1)} = \dfrac{2x^2}{x-1}$ $\quad x \neq 0, 1$

(9) $|3-x| = 4$

Remember $|3-x| = |x-3|$

(a) $|x-3| = 4$

x's distance from 3 is 4



$\{-1, 7\}$

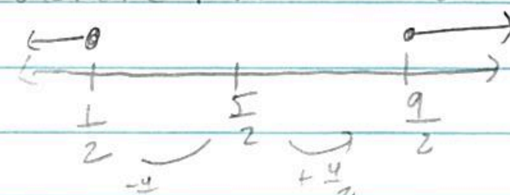
$|5-2x| \geq 4$ $\quad * \ |5-2x| = |2x-5|$

(b) $|2x-5| \geq 4$

$2\left|x-\frac{5}{2}\right| \geq 4$

$\left|x-\frac{5}{2}\right| \geq \frac{4}{2}$

x's distance from $5/2 \geq \frac{4}{2}$



$\{x \mid x \leq 1/2 \text{ or } x \geq 9/2\}$ set builder

$(-\infty, \frac{1}{2}] \cup [\frac{9}{2}, \infty)$ interval

(10) (a) $\dfrac{4-x^{-2}}{2x^{-1}-x^{-2}} = \dfrac{4-\dfrac{1}{x^2}}{\dfrac{2}{x}-\dfrac{1}{x^2}} \cdot \dfrac{x^2}{x^2}$

$\dfrac{4x^2-1}{2x-1} = \dfrac{(2x+1)\cancel{(2x-1)}}{\cancel{(2x-1)}} = 2x+1$ $\quad x \neq 0, \frac{1}{2}$

(10) (b) $\dfrac{x^2-xy}{xy+2y^3} \div \dfrac{x^2+xy}{xy+y^2}$

$y \neq 0,\ x \neq 0$

$x \neq -2y^2, -y$

$\dfrac{\cancel{x}(x-y)}{\cancel{y}(x+2y^2)} \cdot \dfrac{\cancel{y}\cancel{(x+y)}}{\cancel{x}\cancel{(x+y)}} = \dfrac{x-y}{x+2y^2}$

(11) $\dfrac{\overset{(x-5)(x+1)}{x^2-4x-5}}{(x-2)^2} > 0$

		-1		2		5	
$(x-5)(x+1)$	$+$		$-$		$-$		$+$
$(x-2)^2$	$+$		$+$		$+$		$+$
Quotient	$+$		$-$		$-$		$+$

always (+)

SB (a) $\{x \mid x < -1 \lor x > 5\}$

IN (b) $(-\infty, -1) \cup (5, \infty)$

(12) $f(x) = -x^2 + 4x + 6$

$f(x) = -(x^2 - 4x + 4 - 4 - 6)$

$f(x) = -(x-2)^2 - (-10)$

$f(x) = -(x-2)^2 + 10$

13-16 on sheet at end of packet

(17) (a) $f(x) = -x^4 + 4x^2$

$f(-x) = -(-x)^4 + 4(-x)^2$

$= -x^4 + 4x^2$

even

(b) $f(x) = \dfrac{x^3}{x^2-4}$

$f(-x) = \dfrac{(-x)^3}{(-x)^2-4} = \dfrac{-x^3}{x^2-4}$

odd

(18)

$$\begin{array}{r} x^2 - 7 \\ x^2+2 \overline{) x^4 + 0x^3 - 5x^2 + 6x - 7} \\ \underline{-x^4 \quad\quad \mp 2x^2} \\ -7x^2 + 6x - 7 \\ \underline{\pm 7x^2 \quad \pm 14} \\ 6x + 7 \end{array}$$

$6x+7$ remainder

(19)

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 0 & -5 & 10 \\ & & 3 & 9 & 27 & 66 \\ \hline & 1 & 3 & 9 & 22 & 76 \end{array}$$

$x^3 + 3x^2 + 9x + 22 + \frac{76}{x-3}$

(20)

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$\therefore$ $(x-3)$ is a factor

$(x-3)(x^2+3x+2)$

$(x-3)\boxed{(x+2)(x+1)}$ other two factors

(21) $f(2) = 2^3 - 13(2)^2 + 23(2) - 11 = 8 - 52 + 46 - 11 = -9$

since $f(2) \neq 0$
$(x-2)$ is not a factor of $f(x)$

(22) $f(x) = -3x^2 + 5x + 4x^3 - 6$

$f(x) = 4x^3 - 3x^2 + 5x - 6$

possible rational zeros: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

$= \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

23

$f(x) = x^3 - 13x - 12$ poss rational zeros: $\frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 12}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 12$

$f(-1) = (-1)^3 - 13(-1) - 12 = -1 + 13 - 12 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$(x^2 - x - 12)(x+1)$

$(x-4)(x+3)(x+1)$ complete factorization

24 $f(x) = x^3 - 13x - 12$

We found the complete factorization in 23, we set that $= 0$ and solve

$(x-4)(x+3)(x+1) = 0$

$x = 4$ | $x = -3$ | $x = -1$ roots are $\{-3, -1, 4\}$

25 16

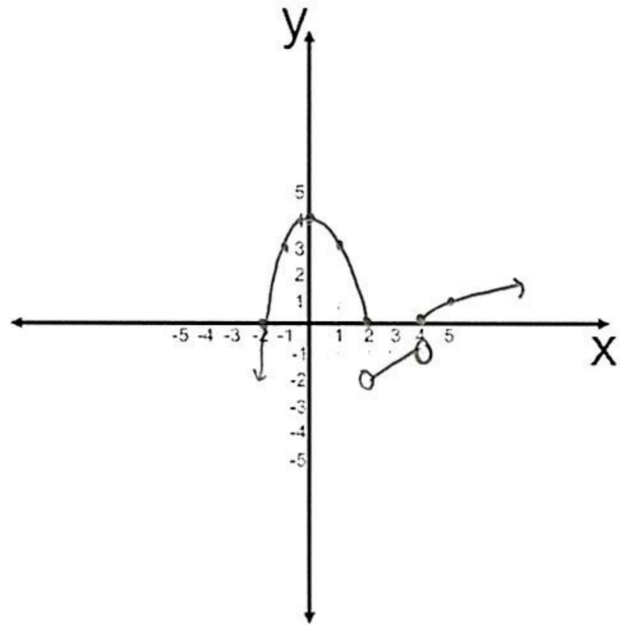
26 $x - 5$

27 and 28 on sheet

12. Write $f(x) = -x^2 + 4x + 6$ in vertex form.

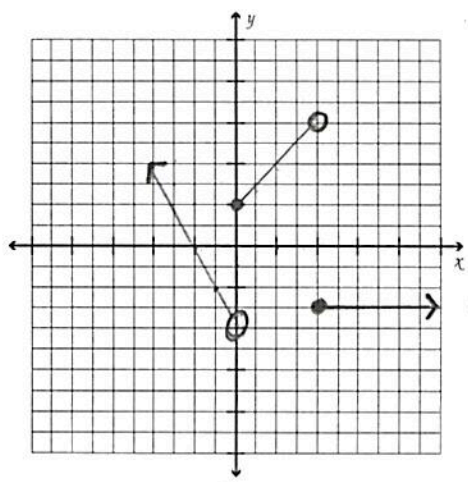
13. Sketch the function **without** using a graphing calculator. Find the domain and range of each function.

a. $f(x) = \begin{cases} -x^2 + 4, & x \le 2 \\ \frac{1}{2}x - 3, & 2 < x < 4 \\ \sqrt{x-4}, & x \ge 4 \end{cases}$



D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

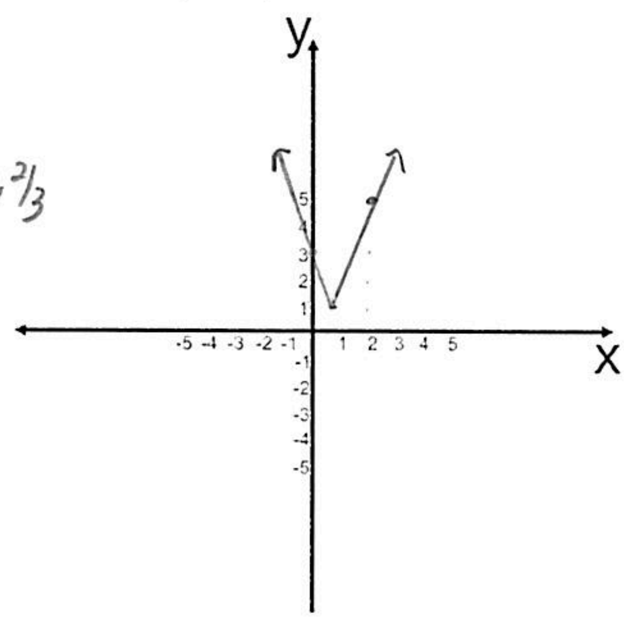
14. Write a piecewise function for the graph



$f(x) = \begin{cases} -2x - 4 & x < 0 \\ x + 2 & 0 \le x < 4 \\ -3 & x \ge 4 \end{cases}$

15. Use the algebraic definition of absolute value to rewrite $f(x) = |3x - 2| + 1$ as a piecewise function and then sketch each graph.

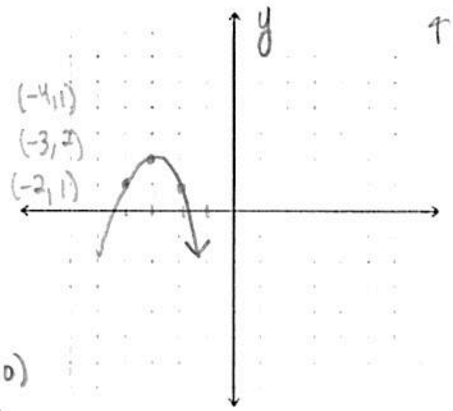
$f(x) = \begin{cases} 3x - 2 + 1 = 3x - 1 \text{ if } 3x - 2 \ge 0,\ x \ge 2/3 \\ -3x + 2 + 1 = -3x + 3 \text{ if } x < 2/3 \end{cases}$



(a) X-int
$0 = 2-(x+3)^2$
$-2 = -(x+3)^2$
$2 = (x+3)^2$
$\pm\sqrt{2} = x+3$
$-3 \pm \sqrt{2} = x$

16. Describe each transformation in terms of the parent function and then graph the function. State the domain, range, and any x- or y- intercepts.

a. $f(x) = 2-(x+3)^2$ left 3, reflect over x, ↑2

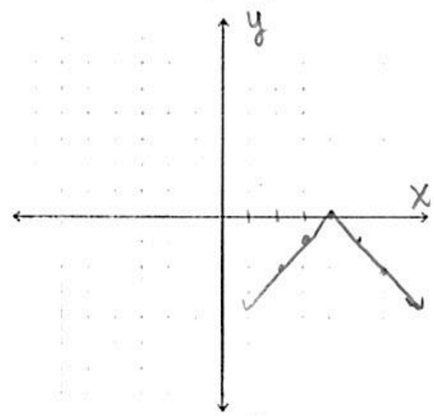


1,1) (-4,1) (-4,-1) (-4,1)
),0) (-3,0) (-3,0) (-3,2)
1,1) (-2,1) (-2,-1) (-2,1)

D: $(-\infty, \infty)$
R: $(-\infty, 2]$
x-int $(-3 \pm \sqrt{2}, 0)$
y-int $(0, -7)$

b. $f(x) = -|x-4|$ right 4, reflect over x

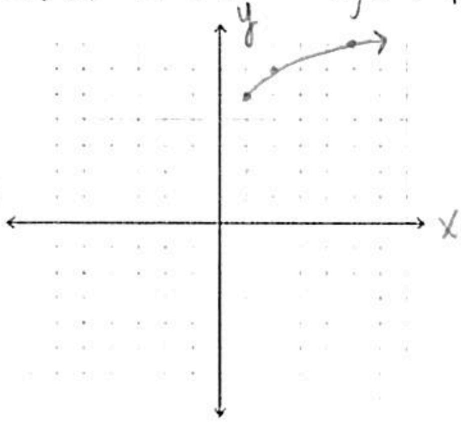
(-1,1) → (3,1) → (3,-
(0,0) (4,0) (4,0
(1,1) (5,1) (5,-



D: $(-\infty, \infty)$
R: $(-\infty, 0]$
x-int (4,0)
y-int (0,-4)

c. $f(x) = \sqrt{x-1}+5$ right 1 ↑5

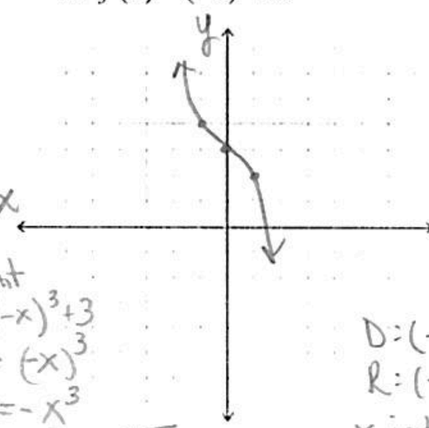
(0,0) → (1,0) → (1,5)
(1,1) (2,1) (2,6)
(4,2) (5,2) (5,7)



D: $[1, \infty)$
R: $[5, \infty)$
X-int: none
y-int: none

d. $f(x) = (-x)^3+3$ reflect over y, ↑3

(-1,-1) → (1,-1) → (1,
(0,0) (0,0) (0,
(1,1) (-1,1) (-1,



X-int
$0 = (-x)^3+3$
$-3 = (-x)^3$
$-3 = -x^3$
$3 = x^3$ $x = \sqrt[3]{3}$

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$
X-int $(\sqrt[3]{3}, 0)$
y-int (0,3)

17. Determine algebraically if the following functions are even, odd, or neither

a. $f(x) = -x^4 + 4x^2$ b. $f(x) = \dfrac{x^3}{x^2-4}$

18. Use polynomial long division to find the quotient of $x^4 - 5x^2 + 6x - 7$ divided by $x^2 + 2$

19. Use synthetic division to find the quotient of $(x^4 - 5x + 10) \div (x-3)$

20. Show that (x-3) is a factor of $P(x) = x^3 - 7x - 6$, and find the other factors.

21. Determine if (x-2) is a factor of $f(x) = x^3 - 13x^2 + 23x - 11$

22. List all of the possible rational roots for $f(x) = -3x^2 + 5x + 4x^3 - 6$

23. What is the complete factorization of $f(x) = x^3 - 13x - 12$

24. What are the roots of $f(x) = x^3 - 13x - 12$

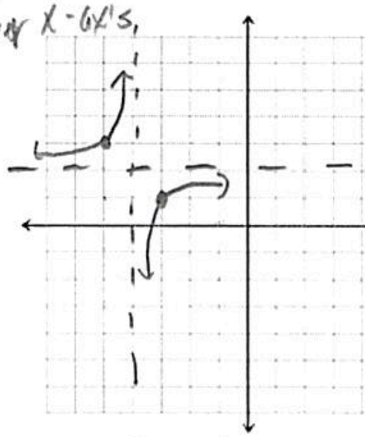
25. If (x-16) is a factor of f(x), then what is one of the zeros?

26. If f(5)=0, what is one of the factors of f(x)?

27. Graph the following using a minimum of 2 points. For each graph, state the domain, range, intercepts, and the equations of any asymptotes.

a. $y = -\dfrac{1}{(x+4)} + 2$

left 4
reflect over x-axis,
↑ 2

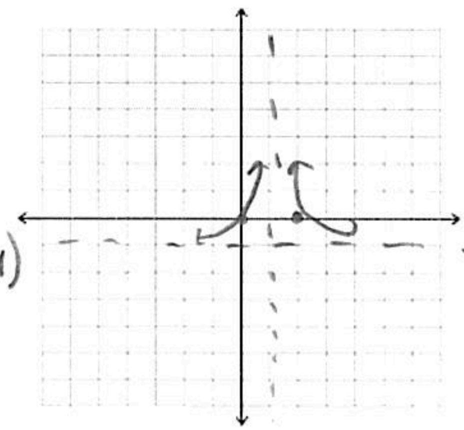


D: $x \neq -4$
R: $y \neq 2$
x-int $(-7/2, 0)$
y-int $(0, 1\tfrac{3}{4})$
VA: $x = -4$
HA $y = 2$

$(1,1) \to (-3,1) \to (-3,-1), (-3,1)$
$(-1,-1) \to (-5,-1) \to (-5,1), (-5,3)$

b. $y = \dfrac{1}{(x-1)^2} - 1$

right 1 ↓1



$(1,1) \to (2,1) \to (2,0)$
$(-1,1) \to (0,1) \to (0,0)$
D: $x \neq 1$
R: $y > -1$
x-int $(0,0), (2,0)$
y-int $(0,0)$
VA $x = 1$
HA $y = 1$

28. Fill in the chart:

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	x-intercept(s)	y-intercept
$y = \dfrac{\overset{-1}{\cancel{5-x}}}{\underset{\cancel{(x-5)}(x+5)}{x^2-25}}$	$(5, -\frac{1}{10})$	$x = -5$	$y = 0$	none	$(0, -\frac{1}{5})$

* x-int for 27a

$0 = -\dfrac{1}{(x+4)} + 2$

$-2 = \dfrac{-1}{(x+4)}$

$2 = \dfrac{1}{x+4}$

$2x + 8 = 1$

$2x = -7$

$x = -7/2$ or -3.5

28

$y = \dfrac{-1}{x+5}$

left 5
reflect over x

y-intercept

$y = \dfrac{-1}{0+5} = \dfrac{-1}{5}$